FAN, TA'LIM VA AMALIYOT INTEGRATSIYASI

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LAGRANJ KO'PAYTUVCHILARI YORDAMIDA XALQARO OLIMPIADA TENGSIZLIKLARINI ISBOTLASH.

Annotatsiya. Xalqaro Matematika Olimpiadalarida tengsizliklarni isbotlash alohida o'rin tutadi. Bu kabi tengsizliklarni isbotlashning turli usullari mavjud bo'lib, ushbu maqolada ishlatilishi sodda va qulay hisoblangan Lagranj ko'paytuvchilari yordamida tengsizliklarni isbotlash keltirilgan.

Kalit so'zlar. Lagranj ko'paytuvchilari, shartli ekstremum, ekstremumning yetarli sharti.

Abstract. Proving inequalities has a important role in the International Mathematical Olympiad. There are various methods of proving such inequalities, and this thesis presents the proof of inequalities using Lagrange multipliers, which are considered simple and convenient to use.

Keywords. Lagrange multipliers, conditional extremum, sufficient condition of extremum.

Аннотация. Доказательство неравенств играет важную роль в Международной математической олимпиаде. Существуют различные методы доказательства таких неравенств, и в данной диссертации представлено доказательство неравенств с использованием множителей Лагранжа, которые считаются простыми и удобными в использовании.

Ключевые слова. множители Лагранжа, условный экстремум, достаточное условие экстремума.

1. Kirish.

Ma'lumki, bir va ko'p o'zgaruvchili funksiyalarning ekstremumlarini topish masalasi amaliyotda dolzarb hisoblanadi. Shuningdek, ko'p o'zgaruvchili funksiyaning ma'lum bir shartlar asosida topilgan ekstremumi optimallashtirish masalalarini yechishda, amaliyotda foydalanishda o'z ahamiyatiga egadir. Xususan, ushbu turdagi funksiyalarning shartli ekstremumlarini topishni matematik olimpiada masalalarida uchraydigan tengizliklarni isbotlashda ham qo'llash mumkin. Quyida shartli ekstremum, Lagranj ko'paytuvchilari haqida tushuncha, shuningdek, ushbu ko'paytuvchilar yordamida Xalqaro Matematik Olimpiada tengsizliklarini isbotlash usuli keltirilgan.

2. Asosiy tushunchalar.

Ta'rif. Faraz qilaylik, n o'zgaruvchili $f(x)$ funksiya a nuqtaning biror atrofida aniqlangan bo'lsin. Agar a nuqtaning shunday atrofi topilib, bu atrofdan olingan istalgan x argument qiymatlari uchun $f(a) \geq f(x)$ $\big(f(a) \leq f(x)\big)$ tengsizlik bajarilsa, u holda f funksiya $a \in \mathbb{R}^n$ nuqtada *lokal maksimum (lokal minimum) ga* ega deyiladi.

Endilikda biz berilgan funksiyaning ekstremal qiymatlarini biror qo'shimcha shartlar bajarilganda topish masalasini o'rganamiz. Bunda asosan qo'shimcha shartlar o'zgaruvchilarning qiymatlarini cheklash shaklida beriladi.

Masalan $n-$ o'zgaruvchili $f(x_1, x_2, \dots, x_n)$ funksiya maksimal yoki minimal qiymatini funksiya argumentlari

$$\varphi_1(x_1, x_2, \dots, x_n) = 0, \varphi_2(x_1, x_2, \dots, x_n) = 0, \dots, \varphi_k(x_1, x_2, \dots, x_n) = 0 \ (n > k)$$

qo'shimcha shartlarni qanoatlantirganda topish masalasini qaraylik. Bunda $\varphi_i(x_1, x_2, \dots, x_n), (\ i = \overline{1, k})$ funksiya ikki marta differensiallanuvchi funksiyadir. Bu holda topilgan maksimum qiymat *shartli maksimum,* minimum qiymat esa *shartli minimum* deyiladi. Shartli maksimum va shartli minimum birgalikda *shartli ekstremum* deb ataladi.

Shartli ekstremumlarni topish uchun asosan *Lagranj ko'paytuvchilar* usulidan foydalaniladi. Ya'ni shunday $\exists \mu_1, \mu_2, \dots, \mu_k$ sonlar tanlab olinib, quyidagi Lagranj funksiyasi tuziladi:

$$L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k) = f(x_1, x_2, \dots, x_n) - \sum_{i=1}^{k} \mu_i \varphi_i(x_1, x_2, \dots, x_n)$$

So‘ngra, Lagranj funksiyasining barcha xususiy hosilalarini nolga tenglashtirishdan hosil bo‘lgan quyidagi tenglamalar sistemasi yechiladi:

$$\begin{cases} \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_1} = 0 \\ \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_2} = 0 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_n} = 0 \\ \varphi_1(x_1, x_2, \dots, x_n) = 0 \\ \varphi_2(x_1, x_2, \dots, x_n) = 0 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \varphi_k(x_1, x_2, \dots, x_n) = 0 \end{cases}$$

Ushbu $(n + k)$ ta noma`lumli tenglamalar sistemasini yechish orqali topilgan $A(x_1, x_2, \dots, x_n)$ nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning shartli ekstremum nuqtasi bo‘ladi. Topilgan nuqtaning shartli maksimum yoki shartli minimum ekanligini aniqlash uchun shartli ekstremumning yetarli shartlaridan foydalaniladi.

Shartli ekstremumning yetarli shartlarini quyidagicha keltirish mumkin:

$A(x_1, x_2, \dots, x_n)$ nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning shartli ekstremum nuqtasi bo‘lsin. $\varphi_i(x_1, x_2, \dots, x_n), i = \overline{1, k}\,, k < n$ funksiyalar A nuqtaning biror atrofida uzluksiz ikkinchi tartibli xususiy hosilalarga ega va quyidagi F matritsaning rangi k ga teng bo‘lsin:

$$F = \begin{pmatrix} \dfrac{\partial \varphi_1(A)}{\partial x_1} & \dfrac{\partial \varphi_1(A)}{\partial x_2} & \cdots & \dfrac{\partial \varphi_1(A)}{\partial x_n} \\ \dfrac{\partial \varphi_2(A)}{\partial x_1} & \dfrac{\partial \varphi_2(A)}{\partial x_2} & \cdots & \dfrac{\partial \varphi_2(A)}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial \varphi_k(A)}{\partial x_1} & \dfrac{\partial \varphi_k(A)}{\partial x_2} & \cdots & \dfrac{\partial \varphi_k(A)}{\partial x_n} \end{pmatrix}.$$

Agar $d^2L(A)$ kvadratik forma $musbat$ $(manfiy)$ aniqlangan bo‘lsa, A nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning $\varphi_i(x_1, x_2, \dots, x_n) = 0, i = \overline{1, k}$ cheklov tenglamalarini qanoatlantirgandagi $shartli\ minimum$ $(shartli\ maksimum)$ nuqtasi bo‘ladi.

Algebra kursidan ma’lumki,

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j, \qquad (a_{ij} = a_{ji}), \qquad a_{ij} \in R\,.$$

kvadratik formaning musbat aniqlangan bo‘lishi uchun kvadratik forma matritsasining bosh minorlari musbatligi, manfiy aniqlangan bo‘lishi uchun esa bosh minorlarining ishoralari almashinib(manfiy ishoradan boshlab) kelishi yetarli.

Biz ko‘nikma hosil qilishimiz uchun dastlab $f(x,y)$ funksiya $\varphi(x,y)=0$ shart bilan berilganda shartli ekstremumga tekshirish masalasini ko‘rib chiqamiz.

Yuqoridagi yetarlilik shartiga ko‘ra, $d^2L(A)$ kvadratik formaning musbat yoki manfiy aniqlanganlikka tekshiramiz. Malumki, $d^2L(A)=f''_x(A)dx^2+2f''_{xy}(A)dxdy+f''_y(A)dy^2$ tenglik o‘rinli. $\varphi(x,y)=0$ tenglikdan $\frac{dx}{dy}=-\frac{\varphi'_y(A)}{\varphi'_x(A)}$ kelib chiqishini e’tiborga olsak, kvadratik formaning ko‘rinishi quyidagicha bo‘ladi:

$$d^2L(A)=dy^2\left(f''(A)\frac{dx^2}{dy^2}+2f''_{xy}(A)\frac{dx}{dy}+f''_y(A)\right)=$$

$$=\frac{dy^2}{(\varphi'_x)^2}\left(f''_x(A)\left(\varphi'_y(A)\right)^2-2f''_{xy}(A)\varphi'_x(A)\varphi'_y(A)+f''_y(A)\left(\varphi'_x(A)\right)^2\right)=$$

$$=-\frac{dy^2}{(\varphi'_x(A))^2}\begin{vmatrix}0 & \varphi'_x(A) & \varphi'_y(A)\\ \varphi'_x(A) & L''_{xx}(A) & L''_{xy}(A)\\ \varphi'_y(A) & L''_{yx}(A) & L''_{yy}(A)\end{vmatrix}.$$

Quyidagicha belgilash kiritadigan bo‘lsak:

$$H(A)=\begin{vmatrix}0 & \varphi'_x(A) & \varphi'_y(A)\\ \varphi'_x(A) & L''_{xx}(A) & L''_{xy}(A)\\ \varphi'_y(A) & L''_{yx}(A) & L''_{yy}(A)\end{vmatrix},$$

$d^2L(A)$ kvadratik formaning ishorasi $H(A)$ determinantning ishorasiga bog‘liq bo‘ladi ya’ni, agar $H(A)>0$ bo‘lsa, $f(x,y)$ funksiya shartli maksimumga, aks holda shartli minimumga erishadi.

Endi $f(x_1,x_2,\dots,x_n)$ funksiyani $\varphi_i(x_1,x_2,\dots,x_n)=0,\ i=\overline{1,k}, k<n$ shartlar bilan berilganda shartli ekstremumga tekshirish masalasini qaraylik. Bu holda ham $d^2L(A)$ kvadratik formaning ishorasi quyidagi H determinantga bog‘liq bo‘ladi:

$$H=\begin{vmatrix} 0 & 0 & \dots & 0 & \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_1}{\partial x_2} & \dots & \frac{\partial\varphi_1}{\partial x_n} \\ 0 & 0 & \dots & 0 & \frac{\partial\varphi_2}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_2} & \dots & \frac{\partial\varphi_2}{\partial x_n} \\ . & . & \dots & . & . & . & \dots & . \\ 0 & 0 & \dots & 0 & \frac{\partial\varphi_k}{\partial x_1} & \frac{\partial\varphi_k}{\partial x_2} & \dots & \frac{\partial\varphi_k}{\partial x_n} \\ \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_1} & \dots & \frac{\partial\varphi_k}{\partial x_1} & \frac{\partial^2 L}{\partial^2 x_1} & \frac{\partial^2 L}{\partial x_1\partial x_2} & \dots & \frac{\partial^2 L}{\partial x_1\partial x_n} \\ \frac{\partial\varphi_1}{\partial x_2} & \frac{\partial\varphi_2}{\partial x_2} & \dots & \frac{\partial\varphi_k}{\partial x_2} & \frac{\partial^2 L}{\partial x_2\partial x_1} & \frac{\partial^2 L}{\partial^2 x_2} & \dots & \frac{\partial^2 L}{\partial x_2\partial x_n} \\ . & . & \dots & . & . & . & \dots & . \\ \frac{\partial\varphi_1}{\partial x_n} & \frac{\partial\varphi_2}{\partial x_n} & \dots & \frac{\partial\varphi_k}{\partial x_n} & \frac{\partial^2 L}{\partial x_n\partial x_1} & \frac{\partial^2 L}{\partial x_n\partial x_2} & \dots & \frac{\partial^2 L}{\partial^2 x_n} \end{vmatrix},$$

Agar H matritsaning $H_{k+1}, H_{k+2}, \dots, H_{k+n}$ minorlarning A nuqtadagi qiymatlari ishorasi $(-1)^k$ ning ishoralari bilan bir xil bo‘lsa, u holda A nuqta shartli minimum nuqta bo‘ladi.

Agar H matritsaning $H_{k+1}, H_{k+2}, \dots, H_{k+n}$ minorlarning A nuqtadagi qiymatlari ishorasi navbat bilan o‘zgarsa va $H_{k+1}(A)$ ning ishorasi $(-1)^{k+1}$ bilan bir xil bo‘lsa, u holda A nuqta shartli maksimum nuqta bo‘ladi. Ushbu determinantni shartli ravishda “yetarlilik determinant” deb ataymiz.

3. Masalaning qo‘yilishi.

Quyida Lagranj ko‘paytuvchilari yordamida yechish mumkin bo‘lgan tengsizliklar keltirilgan.

1-misol. Quyidagi tengsizlikni isbotlang:

$$\frac{x^n+y^n}{2} \geq \left(\frac{x+y}{2}\right)^n$$

bu yerda $n \geq 1$ va $x \geq 0, y \geq 0$.

Isbot. Dastlab quyidagicha belgilash kiritaylik: $x+y=a,\ a \in R^+$.

Masala shartiga ko‘ra, $f(x,y)=\frac{x^n+y^n}{2}$ funksiyaning argumentlari

$$\varphi(x,y)=x+y-a=0,$$

shartni qanoatlantirganda shartli minimum qiymatini topishimiz yetarli.

Lagranj funksiyasini tuzamiz:

$$L(x,y,\mu)=\frac{x^n+y^n}{2}-\mu(x+y-a),$$

bu yerda $x, y \in \mathbb{R}^+$.

Tuzilgan Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \dfrac{\partial L(x,y,\mu)}{\partial x} = \dfrac{nx^{n-1}}{2} - \mu = 0 \\ \dfrac{\partial L(x,y,\mu)}{\partial y} = \dfrac{ny^{n-1}}{2} - \mu = 0 \\ x + y - a = 0 \end{cases}$$

Bu tenglamalar sistemasidan $x = y = \frac{a}{2}$ tenglikni aniqlaymiz. Topilgan $A\left(\frac{a}{2}, \frac{a}{2}\right)$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun yuqorida ta`rifi keltirilgan "yetarlilik determinanti"ning $A\left(\frac{a}{2}, \frac{a}{2}\right)$ nuqtadagi ishorasini aniqlaymiz. Buning uchun dastlab quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x = 1, \ \ \varphi'_y = 1;$

2) $L''_{xx}|_A = \left.\dfrac{n(n-1)x^{n-2}}{2}\right|_A = \dfrac{n(n-1)a^{n-2}}{2^{n-1}} > 0, \quad L''_{xy} = L''_{yx} = 0,$

$L''_{yy}|_A = \left.\dfrac{n(n-1)y^{n-2}}{2}\right|_A = \dfrac{n(n-1)a^{n-2}}{2^{n-1}} > 0.$

Agar $\frac{n(n-1)a^{n-2}}{2^{n-1}} = b > 0$ almashtirish olsak, u holda "yetarlilik determinant" quyidagi ko'rinishda bo'ladi:

$$H|_A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & b & 0 \\ 1 & 0 & b \end{vmatrix} = -2b < 0.$$

Demak, A nuqtada $f(x,y)$ funksiya shartli minimumga erishadi.

Bundan esa berilgan tengsizlik kelib chiqadi:

$$f(x,y) = \frac{x^n + y^n}{2} \geq \frac{1}{2}\left(\left(\frac{a}{2}\right)^n + \left(\frac{a}{2}\right)^n\right) = \left(\frac{a}{2}\right)^n = \left(\frac{x+y}{2}\right)^n.$$

2-misol [Gabriel Dospinescu, Marian Tetiva]. Agar $x, y, z \in R^+, x + y + z = xyz$ bo'lsa, quyidagi tengsizlikni isbotlang:

$$(x-1)(y-1)(z-1) \leq 6\sqrt{3} - 10.$$

Isbot. Masala shartiga ko'ra, $f(x,y,z) = (x-1)(y-1)(z-1)$ funksiyaning argumentlari $\varphi(x,y,z) = xyz - x - y - z = 0$ shartni qanoatlantirganda shartli maksimum qiymatini topish yetarli.

Dastlab, Lagranj funksiyasini tuzib olamiz:

$$L(x,y,z) = (x-1)(y-1)(z-1) + \mu(xyz - x - y - z).$$

Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini yechamiz:

$$\begin{cases} \dfrac{\partial L(x,y,z,\mu)}{\partial x} = (y-1)(z-1) + \mu(yz-1) = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial y} = (x-1)(z-1) + \mu(xz-1) = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial z} = (x-1)(y-1) + \mu(xy-1) = 0 \\ xyz - x - y - z = 0 \end{cases}$$

Bu tenglamalar sistemasini yechib, quyidagilarni topamiz:

$$x = y = z = \sqrt{3}, \quad \mu = -\frac{(\sqrt{3}-1)^2}{2}.$$

Topilgan $A(\sqrt{3}, \sqrt{3}, \sqrt{3})$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun shartli ekstremumning yetarlilik shartiga tekshiramiz. Dastlab, quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x = (yz-1)|_A = 2,\ \ \varphi'_y = (xz-1)|_A = 2,\ \ \varphi'_z = (xy-1)|_A = 2;$

2) $F''_{xx} = F''_{yy} = F''_{zz} = 0,\ \ F''_{xy} = F''_{yx} = (z-1+\mu z)|_A = \frac{(\sqrt{3}-1)^2}{2},$

$F''_{xz} = F''_{zx} = (y-1+\mu y)|_A = \dfrac{(\sqrt{3}-1)^2}{2},\ \ F''_{yz} = F''_{zy} = (x-1+\mu x)|_A$

$= \dfrac{(\sqrt{3}-1)^2}{2}.$

Agar $\frac{(\sqrt{3}-1)^2}{2} = a$ almashtirish olsak:

$$H_2(A) = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4 < 0, \quad H_3(A) = \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & a \\ 2 & a & 0 \end{vmatrix} = 8a > 0$$

$$H_3(A) = \begin{vmatrix} 0 & \varphi'_x & \varphi'_y & \varphi'_z \\ \varphi'_x & F''_{xx} & F''_{xy} & F''_{xz} \\ \varphi'_y & F''_{yx} & F''_{yy} & F''_{yz} \\ \varphi'_z & F''_{zx} & F''_{zy} & F''_{zz} \end{vmatrix}_A = \begin{vmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & a & a \\ 2 & a & 0 & a \\ 2 & a & a & 0 \end{vmatrix} = -12a^2 < 0.$$

Demak, $f(x,y,z)$ funksiya A nuqtada shartli maksimumga erishadi. Bundan esa berilgan tengsizlikni hosil qilamiz:

$$(x-1)(y-1)(z-1) \le 6\sqrt{3} - 10.$$

3-misol. Agar x, y – natural sonlar bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\sqrt[x+y]{x^y y^x} \le \frac{x+y}{2}.$$

Isbot. Dastlab quyidagicha belgilash kiritamiz: $x + y = a \in \mathbb{N}$.

Masala shartiga ko‘ra, $f(x,y) = \sqrt[a]{x^y y^x}$ funksiyaning argumentlari

$\varphi(x,y) = x + y - a = 0$ shartni qanoatlantirganda shartli maksimum qiymatini topishimiz kerak. Lagranj funksiyasini tuzib olamiz:

$$L(x,y,\mu)=\sqrt[a]{x^y y^x}-\mu(x+y-a).$$

Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini tuzamiz:

$$\begin{cases}\dfrac{\partial L(x,y,\mu)}{\partial x}=\dfrac{y}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\dfrac{lny}{a}x^{\frac{y}{a}}y^{\frac{x}{a}}-\mu=0\\ \dfrac{\partial L(x,y,\mu)}{\partial y}=\dfrac{x}{a}x^{\frac{y}{a}-1}y^{\frac{x}{a}}+\dfrac{lnx}{a}y^{\frac{x}{a}}x^{\frac{y}{a}}-\mu=0\\ x+y-a=0\end{cases}$$

Bu tenglamalar sistemasining yechimi $x=y=\frac{a}{2}$ bo‘lishini aniqlash qiyin emas.

Topilgan $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun quyidagi “yetarlilik determinant”ning $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtadagi ishorasini aniqlaymiz. Quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x=1,\quad \varphi'_y=1;$

2) $L''_{xx}|_A=\left(\dfrac{y}{a}\left(\dfrac{y}{a}-1\right)x^{\frac{y}{a}-2}y^{\frac{x}{a}}+\dfrac{2ylny}{a^2}x^{\frac{y}{a}-1}y^{\frac{x}{a}}+\dfrac{ln^2y}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}}\right)\Bigg|_A=$

$$=-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a},$$

$L''_{xy}|_A=L''_{yx}|_A$

$$=\left(\frac{1}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\frac{y}{a}\left(\left(\frac{x}{a}-1\right)y^{\frac{x}{a}-2}x^{\frac{y}{a}}+\frac{lnx}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}\right)\right.+\frac{lnxlny}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}}$$

$$\left.++\frac{xlny}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}-1}\right)|_A=\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a},$$

$L''_{yy}|_A=\left(\dfrac{x}{a}\left(\dfrac{x}{a}-1\right)y^{\frac{x}{a}-2}x^{\frac{y}{a}}+\dfrac{2xlnx}{a^2}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\dfrac{ln^2x}{a^2}y^{\frac{x}{a}}x^{\frac{y}{a}}\right)\Bigg|_A=$

$$=-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}.$$

Agar $-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}=b$ va $\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}=c$ almashtirishlarni olib, $c-b=\frac{1}{a}$ tenglikni e`tiborga olsa, u holda “yetarlilik determinanti” quyidagi ko‘rinishda bo‘ladi:

$$H|_A=\begin{vmatrix}0&\varphi'_x&\varphi'_y\\ \varphi'_x&L''_{xx}&L''_{xy}\\ \varphi'_y&L''_{yx}&L''_{yy}\end{vmatrix}_A=\begin{vmatrix}0&1&1\\1&b&c\\1&c&b\end{vmatrix}=2(c-b)=\frac{2}{a}>0$$

Demak, $f(x,y,z)$ funksiya A nuqtada shartli maksimumga erishadi. Bundan esa quyidagi munosabat kelib chiqadi:

$$f(x,y) = \sqrt[x+y]{x^y y^x} \le \sqrt[a]{\left(\frac{a}{2}\right)^{\frac{a}{2}} \left(\frac{a}{2}\right)^{\frac{a}{2}}} = \frac{a}{2} = \frac{x+y}{2}.$$

Xulosa.

Maktab o‘quvchilari o‘rtasida o‘tkaziladigan turli Matematika Olimpiadalarida, xususan, Xalqaro Matematika Olimpiada(IMO) masalalarida tengsizliklarni isbotlashda yuqoridagi kabi Lagranj ko‘paytuvchilaridan foydalanish qulay va bir muncha soddadir. Ko‘rinib turibdiki, ushbu tushuncha elementar matematika masalalarini Oliy matematika yordamida hal qilish imkonini beradi.

Foydalanilgan adabiyotlar:

1. Alimov Sh., Ashurov R. Matematik tahlil. 2-qism. “Mumtoz so‘z”, Toshkent, 2018.

2. Т.Азларов., Ҳ.Мансуров. Математик анализ 2-қисм. “Ўқитувчи” , Тошкент 1989.

3. Тер-Крикоров А.М. , Шабунин.М.И. Курс математического анализа: Учеб.пособие для вузов. – 3 -е изд.,изправл. –М.:ФИЗМАТ-ЛИТ, 2001.

4. Б.П.Демидович. Сборник задач и математическому анализу. “Наука”, Москва 1972.

5. https://tutorial.math.lamar.edu/classes/calciii/lagrangemultipliers

6.https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Calculus_(OpenStax)/14%3A_Differentiation_of_Functions_of_Several_Variables

7. https://mathworld.wolfram.com/LagrangeMultiplier.html